Generally Covariant Maxwell Theory for Media with a Local Response: Progress since 2000

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***Abstract*—In the recent decades, it became more and more popular for engineers, physicists, and mathematicians alike to put the Maxwell equations into a generally covariant form. This is particularly useful for understanding the fundamental structure of electrodynamics (conservation of electric charge and magnetic flux). Moreover, it is ideally suited for applying it to media with local (and mainly linear) response behavior. We try to collect the new knowledge that grew out of this development. We would like to ask the participants of EMTS 2016 to inform us of work that we may have overlooked in our review.**

Premetric classical electrodynamics has been consistently formulated in 2003 in the book of Hehl and Obukhov [18], see there for earlier references. We may add also O'Dell [65] and Serdyukov et al. [75]. This framework consists of the generally covariant Maxwell equations in the 4-dimensional or the (1+3)-decomposed version, see Fig.1:

$$dH = J \quad \begin{cases} \underline{d}\,\mathcal{D} = \rho\,, \\ \dot{\mathcal{D}} = \underline{d}\,\mathcal{H} - j\,. \end{cases} \tag{1}$$

$$dF = 0 \quad \begin{cases} \underline{d}\,B = 0\,, \\ \dot{B} = -\underline{d}\,E\,. \end{cases} \tag{2}$$

If the medium has a local and linear response behavior, the excitation $H = (\mathcal{H}, \mathcal{D})$ is local and linear in the field strength $F = (E, B)$. In components we have

$$\check{H}^{ij} = \frac{1}{2}\chi^{ijkl}\,F_{kl} \quad \text{with} \quad \chi^{ijkl} = -\chi^{jikl} = -\chi^{ijkl}\,. \tag{3}$$

Here $i, j, ... = 0, 1, 2, 3$ and $\check{H}^{ij} := (1/2)\epsilon^{ijkl}H_{kl}$. The response tensor density χ has 36 independent components. It can be decomposed in three irreducible pieces with $36 = 20{+}15{+}1$ components, respectively. Split in (1+3) dimensions, with $a, b, .. = 1, 2, 3$, we have (for details, see [18], [24])

$$D^a = \left(\varepsilon^{ab} - \epsilon^{abc}\,n_c\right)E_b + \left(\gamma^a{}_b + s_b{}^a - \delta^a_b s_c{}^c\right)B^b + \alpha\,B^a\,,$$

$$H_a = \left(\mu^{-1}_{ab} - \epsilon_{abc}m^c\right)B^b + \left(-\gamma^b{}_a + s_a{}^b - \delta^b_a s_c{}^c\right)E_b - \alpha\,E_a\,.$$

Up to here, the metric of spacetime, that is, the gravitational potential, did not enter anywhere. We have a premetric framework. The different irreducible parts split as follows (see [45]):

20 components of principal part: $6\;\varepsilon^{ab}\,,\;6\;\mu^{-1}_{ab}\,,\;8\;\gamma^a{}_b\,;$
15 components of skewon part: $9\;s_a{}^b\,,\;3\;n_c\,,\;3\;m^c\,;$
1 component of axion part: $\alpha\,.$

Progress has been achieved since then by different groups, see the more recent books of Delphenich [11], Favaro [13] (Ph.D. thesis), Lindell [46], [47], Raab & de Lange [69], and Russer [72], see also [6]. Premetric electrodynamics turned out to be a lively and active subject. Mainly the following aspects were developed (we order them roughly chronologically):

1) *Magnetic charge is alien to the Maxwellian structure:* A magnetic charge can be mathematically included in premetric electrodynamics in a consistent way, see Edelen [12], Kaiser [41], and Hehl & Obukhov [20]. However, this structure is alien to the Maxwellian scheme, as long as we insist on electric charge conservation.
2) *Skewon part of the electromagnetic response tensor:* The general concept of the skewon part of the response tensor was introduced in [17], for its relation to dissipation, see [18]. The effects of the skewon on light propagation were studied by Itin, Ni, Obukhov, et al. [62], [34], [35], [53], [58], [32], [33], [56]. A theory of the skewon in interaction with the gravitational field was provided in [23].
3) *Quantum Hall effect is independent of the external gravitational field:* The QHE can be phenomenologically described in a $(1+2)$-dimensional premetric electrodynamic framework; also the local, linear response tensor turns out to be premetric. Therefore, the QHE cannot depend on an external gravitational field, see [19].
4) *Electromagnetism can couple to a possible Cartan torsion of spacetime only nonminimally:* Solanki, Preuss, et al. [68], [76] pointed out that a non-minimal coupling of gravity to electromagnetism, in particular to the torsion field, is possible, see also Hehl and Obukhov [16], Rubilar et al. [71], and Itin et al. [25], [38].
5) *Signature of the metric can be derived from electrodynamics:* In [18] it has been pointed out that the signature of the metric can be derived from the Lenz rule and the positivity of the energy of the electromagnetic field. The formalism has been straightened out, improved, and enriched by Itin et al. [37], [39].
6) *Four incarnations of the Tellegen gyrator:* The Tellegen gyrator [43], [72], [77], [78] is an element in linear and passive network theory, which "rotates" currents I into voltages V. Similarly, (i) the axion part α of the elec-

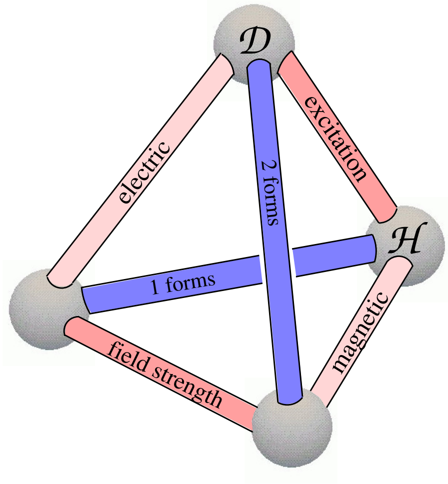

Fig. 1. The electromagnetic field: The interrelation between the excitation $(\mathcal{D}, \mathcal{H})$ and the field strength (E, B).

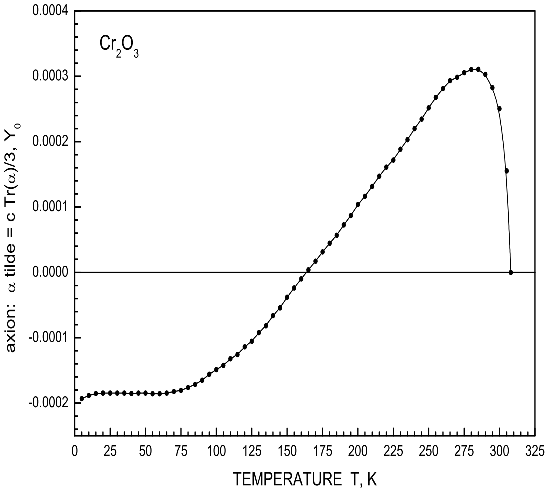

Fig. 2. The pseudoscalar or axion piece $\widetilde{\alpha}$ of the constitutive tensor $\chi^{\lambda\nu\sigma\kappa}$ of Cr_2O_3 in units of $Y_0 = 1/Z_0$ as a function of the temperature T in *kelvin*; here Z_0 is the vacuum impedance which, in SI, is ≈ 377 *ohm*.

tromagnetic response tensor rotates excitations $(\mathcal{H}, \mathcal{D})$ into field strengths (B, E) [18, Eqs.(D.1.112/3)], and so does (ii) the PEMC (perfect electromagnetic conductor) of Lindell & Sihvola [51], [52], see [47, Eq.(7.72)]. (iii), in the QHE, the current/charge (j, ρ) is rotated into the field strength (E, B), see [18, Eq.(B.4.64)].[1] (iv) The hypothetical axion particle of elementary particle physics, see Wilczek [81] and Weinberg [80, pp. 458–461] has the same behavior as (i) and (ii). In the gyrator and additionally, in its four incarnations, the P-symmetry (parity) and the T-symmetry (time) are always violated. It has been experimentally confirmed [24], see Fig.2, that in single crystals of Chromium Sesquioxide Cr_2O_3 the axion part is non-vanishing and maximally of the order of 10^{-3} × vacuum admittance. This falsifies the so-called Post constraint.

7) *Axion and light propagation:* Ni [54], [55] proposed, as a device for violation the equivalence principle, to modify classical electrodynamics by introducing an axion field. Carroll et al. [7] used an axion field with a time-like gradient for violating Lorentz and parity invariance. This model was extended and reformulated in the premetric formalism by Itin [26], [30], [31]. Various theoretical issues of this model were studied by Balakin [4], and Noble [60]. Recently Ni [59], [57] provided constraints on axion and dilaton from polarized/unpolarized laboratory/astrophysical/cosmic experiments/observations.
8) *Forbid birefrinence in vacuo and find the light cone:* Wave propagation in electrodynamics with a local and linear response law exhibits the phenomenon of birefringence. In the geometric optics approximation, this is manifest in the fourth order Fresnel equation for the wave covector. Such a picture is typical for complex (anisotropic, magnetoelectric, moving) media. However, it is natural to assume that the physical vacuum is a non-birefringent continuum. This no-birefringence condition reduces the general Fresnel equation to the light cone [21], [44]; thus, the spacetime metric is recovered from the constitutive tensor. More recent developments can be found in [8], [14], [27].
9) *Dimensional analysis of physical quantities:* In [18] we analyzed electrodynamics in terms of a consistent theory of physical dimension. In particular the operational verification of the physical quantities were at center stage. In [22], we extended these considerations in the context of a discussion with Okun. The vacuum admittance $\sqrt{\varepsilon_0/\mu_0} = e^2/(2h\alpha)$ can be expressed in terms of the fine structure constant α, the elementary charge e and Planck's constant h. The possible time variability of the vacuum impedance and the fine structure constant were discussed by Tobar [79] and us.
10) *New initial value formulation of Maxwell's theory by Perlick:* It was studied by Perlick [66] for the premetric version. He derived several conditions for the evolution equations to be hyperbolic, strongly hyperbolic, or symmmetric hyperbolic. In particular, he characterized all response laws for which the evolution equations are symmetric hyperbolic. The latter property is sufficient, but not necessary, for well-posedness of the initial-value problem. Symbol and *hyperbolic polynomials* of the wave equation and their relation to premetric electrodynamics were discussed by Beig [5]. Itin [29] derived premetric covariant jump conditions for the initial value hypersurface, the light cone, and a boundary between two media.
11) *The Kummer tensor of electrodynamics, Fresnel wave surfaces unmasked as Kummer surfaces:* We found a

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[1] In spite of the last mentioned fact, one cannot build a gyrator out of the QHE, as pointed out to us by von Klitzing [42].

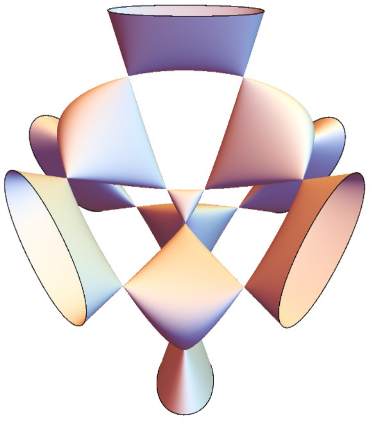

Fig. 3. The Fresnel surface for a metamaterial with the moduli displayed in (5). This surface turns out to be a Kummer surface of algebraic geometry.

fourth rank Kummer tensor cubic in the response tensor,

$$\mathcal{K}^{ijkl} := \chi^{aibj}\, {}^{\diamond}\chi^{\diamond}_{acbd}\, \chi^{ckdl}\,, \tag{4}$$

where $^\diamond$ is a premetric duality operator [3]. The Fresnel surfaces are defined by $\mathcal{K}^{ijkl}\, q_i q_j q_k q_l = 0$; here q_i is the wave covector of the propagating light. Following earlier ideas of Bateman, Favaro et al. [15] were able to show that, for vanisking skewon part, the Fresnel surfaces turn out to be Kummer surfaces with up to 16 singular points, see Fig. 3, where the Fresnel surface of a metamaterial is depicted with the electromagnetic moduli [15]

$$\begin{aligned} \varepsilon^{ab} &= \tfrac{1}{4}\varepsilon_0\, \mathrm{diag}\big(-1-\sqrt{3}, -1-\sqrt{3}, -4+2\sqrt{3}\big), \\ \mu_{ab}^{-1} &= \tfrac{1}{4}\mu_0^{-1}\, \mathrm{diag}\big(1+\sqrt{3}, 1+\sqrt{3}, 4-2\sqrt{3}\big), \\ \gamma^a{}_b &= \tfrac{1}{4}\sqrt{\varepsilon_0\mu_0^{-1}}\, \mathrm{diag}\big(3+\sqrt{3}, -3-\sqrt{3}, 0\big). \end{aligned} \tag{5}$$

Russer pointed out to us [73] that the 16-fold symmetry of the Fresnel-Kummer surface reminds him of a 16-fold solution manifold in the transmission-line matrix (TLM) method, see [72, Sec.14.3] and [74]. Since both phenomena refer to the propagation of electromagnetic signals, there may very well exist an interrelationship.

12) *Possible factorisations of the Fresnel equation (FE):* Which media give rise to such simplifications? Four cases were solved: (i) The FE cannot be factorized in polynomials of lower degree. Materials with anisotropic permittivity and permeability, e.g., display this behavior [28]. (ii) The FE factorizes in irreducible quadratic polynomials. For research on the situation where the two factors are distinct and have Lorentzian signature, see [9], [48], [63]. Light then propagates along two distinct light cones. When the two quadratic factors are the same, up to a constant, a single light cone is obtained, so that there is no birefringence, see 8). (iii) The FE factorizes in four linear polynomials. Extreme Magneto-Electric (EME) materials [50] are known to have this property. (iv) The FE vanishes trivially, i.e., it is satisfied by any wave covector $q_i = (\omega, -k_a)$. Media corresponding to this scenario are described in [49].
13) *Photon propagator in premetric electrodynamics:* The Green tensor is the photon propagator in momentum space. It is important in quantum field theoretical extension of electrodynamics. It was derived and applied to axion and skewon modified electrodynamics by Itin [36], [30], [35]. In a similar way, Andrianov et al. [1] studied the propagation of photons and massive vector mesons in the presence of a medium, which violates Lorentz and a CPT invariance. Pfeifer and Siemssen [67] applied the premetric photon propagator for studying the causal structure and the quantization problems in electrodynamical models with linear response.
14) *Accelerated reference frames in coordinate free form:* Auchmann and Kurz [2], by using a fiber bundle formalism, developed a general theory for observers by splitting 4-dimensional electrodynamics in a covariant way into time and space. This is particularly useful for interpreting experiments in accerelerated motion.
15) *Energy-momentum tensor of the electromagnetic field [61], [70]:* The form of the energy-momentum tensor of the electromagnetic field in a medium with nontrivial electric and magnetic properties is a problem that dates back to the century-old work of Minkowski and Abraham. The Minkowski vs. Abraham controversy can be resolved by the crucial observation that the electromagnetic field in a medium is an open physical system. The closed system of interacting electromagnetism plus matter is characterized by the total energy-momentum tensor that is defined unambiguously within the Lagrange-Noether framework. Both Minkowski and Abraham tensors may arise under appropriate conditions for special splits of the total energy-momentum tensor.
16) *Nonlinear electrodynamics:* So far, we only treated linear response laws. A nonlinear law naturally arises when the quantized electromagnetic field excites radiative vacuum corrections. Alternatively, one can consider a fundamentally nonlinear electrodynamics with a Born-Infeld type Lagrangian, see [18]. Wave propagation in nonlinear models manifest itself typically in the birefringence property. An analysis of geometric optics in nonlinear electrodynamics reveals the product structure of the Fresnel wave surface, which gives rise to two optical metrics, see [64]. It is also possible to formulate a Finsler electrodynamics in a premetric way, see [10] and [40].

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